Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The finite discontinuities occur in both cases for the parameters $\alpha, \beta = \pm 1$.

Also solved by M. E. Graber, G. W. Greenwood, F. D. Posey, S. A. Corey, G. B. M. Zerr, and J. O. Mahoney.

MECHANICS.

170. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Prove that the moment of inertia of an ogival head rotating about its geometrical axis is $\frac{\pi w}{g}\int_0^{R\sqrt{(4n-1)}} y^4 dx$, where w is the weight in pounds of a cubic foot of material, R the radius of the base of the ogive, and n the diameter of projectile.

Solution by LYTLE BROWN, Lehigh University, and the PROPOSER.

An ogival head is one-half the solid generated by the revolution of a segment of a circle about its chord. The equation of the generating curve is $y = \sqrt{(4n^2R^2 - x^2)} - (2n-1)R$, the origin being the center of the base of the ogive. If $y=0$, the resulting value of x is the length of the head, that is, the length of the head is $R\sqrt{(4n-1)}$. The moment of inertia of an elementary cylinder about the axis of x is $4\pi y^2 \frac{\omega}{g}.y^2dx$, or $dI = \frac{\pi\omega}{2g}y^4dx$.

$$\therefore I = \frac{\pi\omega}{g}\int_0^{R(\sqrt{4n}-1)} y^4 dx.$$

AVERAGE AND PROBABILITY.

155. Proposed by E. B. WILSON, Ph. D., Yale University.

The game of craps is played with two dice. If the player throws 7 or 11 on the first throw he wins. If he throws 12, 2, or 3 he loses. If the player throws any other number, that is to say, 4, 5, 6, 8, 9, 10, he is obliged to continue throwing until he throws that number again or until he throws 7. If he succeeds in throwing his first throw before he does 7, he wins—otherwise he loses. Required the odds against him. (Note that he can continue throwing indefinitely without getting either his original throw or the 7).

Remark. A correct solution of this problem is given by Dr. Zerr, in Vol. X, No. 3, p. 81. Mr. J.E. Sanders sent in a different solution, his answer being $\frac{244}{495}$, in favor of the player.

Problem 131 should be numbered 158. Three solutions of this problem have been received, none of which agree.

Problem 157 should be numbered 159,

MISCELLANEOUS.

144. Proposed by IRA M. DeLONG, Professor of Mathematics in the University of Colorado, Boulder, Col.

Determine the number of distinct spherical polygons of n sides formed by arcs of n given great circles on a sphere, each arc to be greater than 0 degrees, and less than 360 degrees, and no two sides of any polygon to lie an the same circle.

Solution by ARNOLD EMCH, Professor of Graphics and Mathematics, University of Colorado.

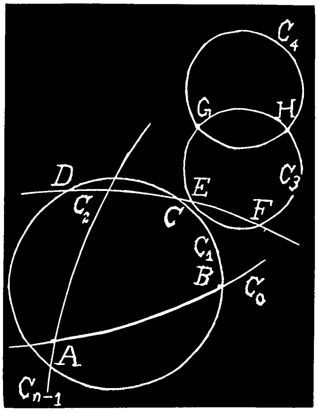
1. No two sides of any of the required polygons must lie on the same circle. Arcs between 0 and 360° are admitted.

2. Designate the n great circles by c_0, c_1, c_2, c_3, ………, c_{n-1}, Take any of the circles, say c_0, and on it all arcs which may be taken as sides of required polygons. Construct all possible polygons of n sides, x in number, each having one of these arcs as a side.

Since each of all possible polygons of n sides on the sphere has one of its sides on c_0, it is clear that the number of these polygons is identical with x.

There are $2(n-1)$ points of intersection of c_0 with the $(n-1)$ remaining circles. These may be combined in $\{2(n-1)[2(n-1)-1]\}/2$ ways to form arcs of the required polygons. But as no two arcs shall be on one circle we have to exclude from this number those arcs which are formed on c_0 by each of the $(n-1)$ circles separately; *i. e.*, $(n-1)$. The number of arcs on c_0, each of which may be taken as a side of a polygon, is therefore $(n-1)[2(n-1)-1]-(n-1)$, or, since we can also take their supplements to 360°,

$$2(n-1)[2(n-1)-1]-2(n-1)=2^2(n-1)(n-2).$$



4. Take now one of these arcs AB and the two circles c_1 and c_{n-1} through B and A. The question is, in how many different ways can I pass from A to B, from B on c_1 to points of arcs of c_2, c_3, ………, c_{n-2}; then on arcs of these c's to points on c_{n-1} and back to A. For this purpose take any of the permutations of the c's, say c_2, c_3, c_4, ………, c_{n-2}, and let c_1 cut c_2 in C and D, c_2 cut c_3 in E and F, c_3 cut c_4 in G and H, ………, c_{n-2} cut c_{n-1} in X and Y, as indicated in the adjoining diagram.

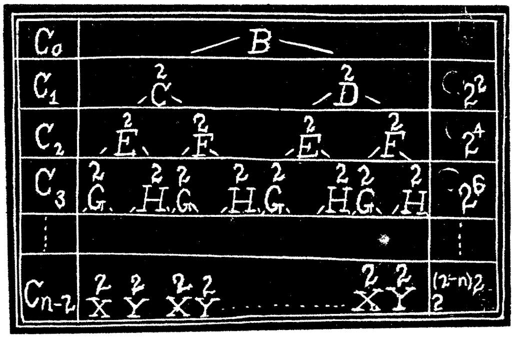
From B, I can reach C and D each in two ways; from C and D, I can, on c_2, reach each of E and F in two ways; so that from B, E, and F may each be reached in 2^3 ways, or both in 2^4 ways. This process continued may be illustrated by the following table.

In this table the first column contains the circles on which the points indicated in the same row are reached. For instance, the points G and H are reached from E and F by describing arcs on the circle c_3. The last column shows in how many ways the points in each row as a whole may be reached from B. Thus all G's and H's in the fourth row may be reached from B in 2^6 different ways. From the table it is seen that the points X and Y which are also on the circle c_{n-1} may be reached from B in 2^{2n-4} different ways. From X and Y each, A may be reached in two ways; hence A may be reached in the prescribed manner in 2^{2n-3} different ways. The same number is obtained for every permutation of c_2, c_3, ………, c_{n-2}; so that there are $(n-3)!2^{2n-3}$ polygons having AB as a side.

5. As there are $2^2(n-1)(n-2)$ different arcs on c_0, the total number x of polygons is

$$2^2(n-1)(n-2)(n-3)!2^{2n-3},$$

$$\text{or } x=(n-1)!2^{2n-1}.$$



6. To find the total number of all possible spherical polygons that may be formed by arcs of n great circles, it is clear that there will be $(r-1)!2^{2r-1}$ polygons of r sides, and consequently

$$\sum_{r=3}^{r=n}[{}^nC_r.(r-1)!2^{2r-1}]$$

will be the number of all possible polygons.

7. *Examples.* With three great circles on a sphere $(3-1)!2^{6-1}=64$ triangles may be formed. The number of spherical triangles and quadrangles which may be formed by four great circles is

$$(4-1)!2^{8-1}+{}^4C_3.(3-1)!2^{6-1}=768+4.64=1024.$$

PROBLEMS FOR SOLUTION.

ALGEBRA.

226. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Find the real roots of the system

$$\begin{aligned} x^2+w^2+v^2&=a^2, & vw+u(y+z)&=bc,\\ w^2+y^2+u^2&=b^2, & wu+v(z+x)&=ca,\\ v^2+u^2+z^2&=c^2, & uv+w(x+y)&=ab. \end{aligned}$$

227. Proposed by G. I. HOPKINS, A. M., Manchester, N. H.

Solve $x+y+xy+x^2y+xy^2+x^3y+2x^2y^2+xy^3+x^3y^2+x^2y^3=11$; $x^4y+3x^3y^2+3x^2y^3+2x^4y^2+4x^2y^3+2x^2y^4+4x^4y^3+4x^3y^4+xy^4+x^5y^2+x^5y^3+2x^4y^4+x^2y^5+x^3y^5=30$.

GEOMETRY.

251. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

Represent the vertices of any regular polygon by the consecutive numbers 1, 2.....p.....q.....r.....n. To find the sides and area of the triangle formed by joining p, q, and r.